

Question 1 continued

Lined writing area with 30 horizontal lines.

(Total 5 marks)

Q1



2.

$$z = 5\sqrt{3} - 5i$$

Find

(a) $|z|$, **(1)**

(b) $\arg(z)$, in terms of π . **(2)**

$$w = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

Find

(c) $\left|\frac{w}{z}\right|$, **(1)**

(d) $\arg\left(\frac{w}{z}\right)$, in terms of π . **(2)**



3.
$$\frac{d^2y}{dx^2} + 4y - \sin x = 0$$

Given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = \frac{1}{8}$ at $x = 0$,

find a series expansion for y in terms of x , up to and including the term in x^3 .

(5)



Question 3 continued

Lined area for writing the answer to Question 3 continued. The area contains 28 horizontal lines.

(Total 5 marks)

Q3



4. (a) Given that

$$z = r(\cos \theta + i \sin \theta), \quad r \in \mathbb{R}$$

prove, by induction, that $z^n = r^n(\cos n\theta + i \sin n\theta)$, $n \in \mathbb{Z}^+$

(5)

$$w = 3\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$$

(b) Find the exact value of w^5 , giving your answer in the form $a + ib$, where $a, b \in \mathbb{R}$.

(2)



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Question 5 continued

(Total 12 marks)

Q5

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Question 6 continued



7. (a) Show that the transformation $y = xv$ transforms the equation

$$4x^2 \frac{d^2y}{dx^2} - 8x \frac{dy}{dx} + (8 + 4x^2)y = x^4 \quad (I)$$

into the equation

$$4 \frac{d^2v}{dx^2} + 4v = x \quad (II) \quad (6)$$

(b) Solve the differential equation (II) to find v as a function of x . (6)

(c) Hence state the general solution of the differential equation (I). (1)



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Question 7 continued

Lined area for writing the answer to Question 7. The page contains 24 horizontal lines.

Q7

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(Total 13 marks)



8.

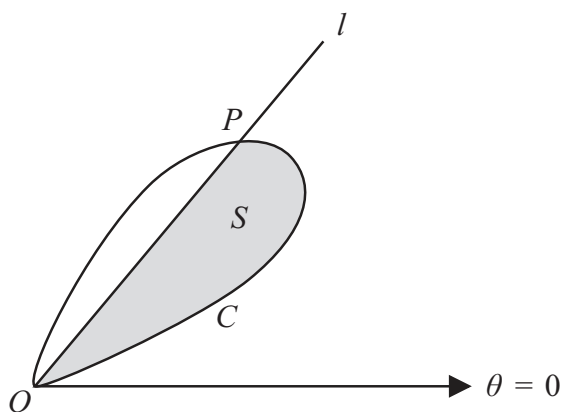


Figure 1

Figure 1 shows a curve C with polar equation $r = a \sin 2\theta$, $0 \leq \theta \leq \frac{\pi}{2}$, and a half-line l .

The half-line l meets C at the pole O and at the point P . The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, ϕ) .

(a) Show that $\cos \phi = \frac{1}{\sqrt{3}}$ **(6)**

(b) Find the exact value of R . **(2)**

The region S , shown shaded in Figure 1, is bounded by C and l .

(c) Use calculus to show that the exact area of S is

$$\frac{1}{36} a^2 \left(9 \arccos \left(\frac{1}{\sqrt{3}} \right) + \sqrt{2} \right)$$
(7)



Question 8 continued

Lined writing area for the answer to Question 8 continued.

Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

